

Effective interactions and shell model studies of heavy tin isotopes

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We present results from large-scale shell-model calculations of even and odd tin isotopes from ^{134}Sn to ^{142}Sn with a shell-model space defined by the $1f_{7/2}, 2p_{3/2}, 0h_{9/2}, 2p_{1/2}, 1f_{5/2}, 0i_{13/2}$ single-particle orbits. An effective two-body interaction based on modern nucleon-nucleon interactions is employed. The shell-model results are in turn analyzed for their pairing content using a generalized seniority approach. Our results indicate that a pairing-model picture captures a great deal of the structure and the correlations of the lowest lying states for even and odd isotopes.

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I. INTRODUCTION

The region of neutron-rich nuclei around ^{132}Sn with the number of protons at or just below the $Z = 50$ shell closure, is an area of current experimental and theoretical interest, see for example Refs. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Recent results span from β and γ spectroscopy at CERN/Isolde of the $N = 82$ r -process “waiting point” nucleus ^{130}Cd [4], with several interesting astrophysical implications, to the measurement of the transition strengths $B(E2; 0^+ \rightarrow 2^+)$ and nuclear moments of Te and Sn isotopes at the HRIBF facility at Oak Ridge National Laboratory [2, 3, 7]. For the latter experiments, the theoretical analysis of Ref. [6], using a quasiparticle random phase approximation, pointed to a weakened pairing strength for the interacting neutrons in order to explain the experimental results.

A series of experiments at CERN/Isolde [11] have aimed at decay studies of $^{135-140}\text{Sn}$ via laser ionization techniques, in order to obtain half-lives and γ -ray spectra for these very neutron-rich Sn nuclides that lie directly in the path of the r -process. In Ref. [8] data were presented for $T_{1/2}$ and neutron emission probabilities for $^{135-137}\text{Sn}$. Recently, data from various β -decays of $^{134,135}\text{Sn}$ were used to confirm and establish new levels in ^{134}Sb and ^{135}Sb [9, 10]. Data for beta-delayed neutron decays of $A = 137$ and $A = 138$ have also been obtained. These and other data are of importance for our understanding of the presence of a single site for the r -process nucleosynthesis, or at least a single mechanism for elements above $Z = 56$ and possibly a second site/mechanism for elements below $Z = 56$, see for example the discussion in Ref. [4].

The lifetime of the isotopes $^{136-138}\text{Sn}$ are of the order of few milliseconds [8, 11], a fact which poses severe limitations to our capability of obtaining spectroscopic information with present laser ionization techniques. Thus, except for the above mentioned ^{134}Sn isotope and the low-energy spectrum of ^{133}Sn [12], data for heavy tin isotopes ($A = 134 - 142$) are either rather limited or just missing. Theoretical analyses, except those reported in Refs. [8, 9, 10] and a study of two and three valence particles by Coraggio *et al* [13], provide little spectroscopic information for nuclei beyond ^{135}Sn . The sole exception we are aware of are the mean-field investigations of Hoffman and Lenske [14] and Dobaczewski *et al* [15]. A theoretical analysis of the spectra of these nuclei can therefore provide several possible guidelines for interpretations of data of relevance for astrophysical studies of the r -process path, and in particular for investigation of spectroscopic trends when one moves towards the neutron drip line.

It is within the latter topic we focus our attention here. The rationale for the present study, in addition to providing spectroscopic information on even and odd Sn isotopes for $A = 134 - 142$, is to analyze the pairing structure of the low-lying states of these nuclei as we move towards the drip line (the reader should note that the drip line for the Sn isotopes is predicted at $A \sim 160$ [15]). The structure of pairing correlations and the strength of the pairing interaction as one moves towards the drip line, in addition to our understanding of how shells evolve, is a topic of great experimental and theoretical interest in low-energy nuclear physics.

To provide the above mentioned information on pairing correlations, we perform large-scale shell-model calculations using realistic effective interactions. We present results from extensive shell model studies of heavy Sn isotopes with up to 10 valence neutrons beyond the ^{132}Sn core. The effective neutron-neutron interaction is tailored for a shell-model space which includes the single-particle states $1f_{7/2}, 2p_{3/2}, 0h_{9/2}, 2p_{1/2}, 1f_{5/2}$ and $0i_{13/2}$. A perturbative many-body scheme is employed to derive the effective interaction starting with a realistic model for the free nucleon-nucleon interaction, as detailed in Ref. [19] and reviewed briefly in the next section. The resulting wave functions of selected low-lying states of odd and even Sn isotopes are in turn analyzed for their pairing content using the generalized seniority approach developed by Talmi [16]. This provides one possible way of extracting pairing correlations and information about the pairing strength as one increases the number of valence nucleons.

This work falls in four sections. In section II we give a brief outline of the theoretical framework for obtaining an effective interaction for the nuclei of interest. Some basic shell-model features are also presented. The results

from shell-model calculations for both even and odd isotopes are presented in section III together with a generalized seniority analysis. Concluding remarks are found in section IV.

II. SHELL-MODEL HAMILTONIAN

We give here a brief review of our calculational recipe, for further details see Ref. [19]. Our scheme to obtain an effective two-body interaction appropriate for heavy tin isotopes starts with a free nucleon-nucleon interaction V appropriate for nuclear physics at low and intermediate energies. Here we employ the potential model of Ref. [20], the so-called charge-dependent Bonn interaction. This is an extension of the one-boson-exchange models of the Bonn group [21], where mesons like π , ρ , η , δ , ω and the fictitious σ meson are included. Thereafter we need to handle the fact that the strong repulsive core of the nucleon-nucleon potential V is unsuitable for perturbative approaches. This problem is overcome by introducing the reaction matrix G given by the solution of the Bethe-Goldstone equation

$$G = V + V \frac{Q}{\omega - H_0} G, \quad (1)$$

where ω is the unperturbed energy of the interacting nucleons, and H_0 is the unperturbed hamiltonian. The operator Q , commonly referred to as the Pauli operator, is a projection operator which prevents the interacting nucleons from scattering into states occupied by other nucleons. In this work we solve the Bethe-Goldstone equation for five starting energies ω , by way of the so-called double-partitioning scheme discussed in for example Ref [19]. The G -matrix is the sum over all ladder type diagrams. This sum is meant to renormalize the strong repulsive short-range part of the interaction. The physical interpretation is that the particles must interact with each other an infinite number of times in order to produce a finite interaction.

Since the G -matrix represents just the summation to all orders of ladder diagrams with particle-particle diagrams, there are obviously other terms which need to be included in an effective interaction. Long-range effects represented by core-polarization terms are also needed. The first step then is to define the so-called \hat{Q} -box given by

$$P\hat{Q}P = PGP + P \left(G \frac{Q}{\omega - H_0} G + G \frac{Q}{\omega - H_0} G \frac{Q}{\omega - H_0} G + \dots \right) P. \quad (2)$$

The \hat{Q} -box is made up of non-folded diagrams which are irreducible and valence linked. We can then obtain an effective two-body interaction $H_{\text{eff}} = \hat{H}_0 + V_{\text{eff}}$ in terms of the \hat{Q} -box, with [19]

$$V_{\text{eff}}(n) = \hat{Q} + \sum_{m=1}^{\infty} \frac{1}{m!} \frac{d^m \hat{Q}}{d\omega^m} \{V_{\text{eff}}(n-1)\}^m, \quad (3)$$

where (n) and $(n-1)$ refer to the effective interaction after n and $n-1$ iterations. The zeroth iteration is represented by just the \hat{Q} -box. Observe also that the effective interaction $V_{\text{eff}}(n)$ is evaluated at a given model space energy ω , as is the case for the G -matrix as well. Here we choose $\omega = -20$ MeV. Less than 10 iterations were needed in order to obtain a numerically stable result. All non-folded diagrams through third order in the interaction G are included. A harmonic oscillator basis was used to derive the single-particle radial wave functions with an oscillator energy $\hbar\omega = 7.87$ MeV. For further details, see Ref. [19].

The present neutron-neutron interaction, the corresponding proton-proton and the proton-neutron interactions derived for holes and particles states using ^{132}Sn as closed shell core, have been employed in several shell-model calculations of nuclei around ^{132}Sn , see for example Refs. [4, 9, 10, 22, 23, 24].

The effective two-particle interaction can in turn be used in shell-model calculations. The shell-model problem requires the solution of a real symmetric $n \times n$ matrix eigenvalue equation

$$\tilde{H} |\Psi_k\rangle = E_k |\Psi_k\rangle, \quad (4)$$

with $k = 1, \dots, K$. At present our basic approach to finding solutions to Eq. (4) is the Lanczo's algorithm; an iterative method which gives the solution of the lowest eigenstates. This method was already applied to nuclear physics problems by Whitehead *et al.* in 1977. The technique is described in detail in Ref. [25] and implemented via the Oslo shell-model code [26]. The eigenstates of Eq. (4) are written as linear combinations of Slater determinants in the m -scheme, distributing the N valence neutrons in all possible ways through the single particle m -scheme orbitals of the model space, $1f_{7/2}$, $2p_{3/2}$, $0h_{9/2}$, $2p_{1/2}$, $1f_{5/2}$ and $0i_{13/2}$. As seen in Table I, the dimensionality n of the eigenvalue matrix \tilde{H} is increasing rapidly with increasing number of valence particles, and for the Sn isotopes of interest can take values up to $n \approx 6 \times 10^7$.

TABLE I: Total number of basis states for the shell-model calculation with the single-particle orbitals $1f_{7/2}, 2p_{3/2}, 0h_{9/2}, 2p_{1/2}, 1f_{5/2}$ and $0i_{13/2}$ defining the model space.

System	Dimension	System	Dimension	System	Dimension
^{134}Sn	62	^{137}Sn	34 804	^{140}Sn	4 606 839
^{135}Sn	448	^{138}Sn	207 514	^{141}Sn	17 574 855
^{136}Sn	4 985	^{139}Sn	1 049 533	^{142}Sn	59 309 576

The single-particle energies are extracted from the experimental ^{133}Sn spectrum [12], except for the $0i_{13/2}$ single-particle orbital which lies above the $^{132}\text{Sn} + n$ threshold 2.45(5) MeV [28]. If one interprets the 2434-keV level in ^{134}Sb as the $(\pi g_{7/2}, \nu i_{13/2})10^+$ configuration [27], the position of the $0i_{13/2}$ orbital can be estimated to 2.6940 ± 0.2 MeV. The adopted single-particle energies used in this work are displayed in Fig. 1.

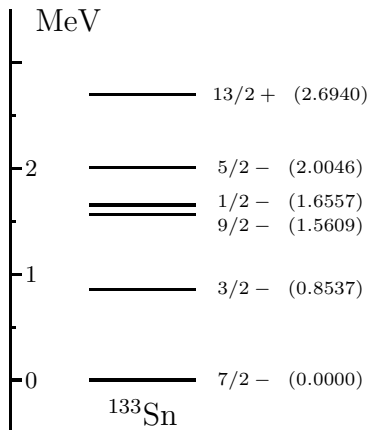


FIG. 1: Adopted single-particle energies for the orbitals $1f_{7/2}, 2p_{3/2}, 0h_{9/2}, 2p_{1/2}, 1f_{5/2}$ and $0i_{13/2}$ in shell model calculations.

We anticipate parts of the discussion of our results by pointing to the fact that there is a difference of 0.85 MeV between the ground state in ^{133}Sn and the first excited state $(3/2)^-$ and 1.56 MeV to the next excited state with spin and parity assignement $(9/2)^-$. Furthermore, the shell gap for neutrons is

$$\begin{aligned}
 \Delta\epsilon_\nu &= \epsilon_\nu(1f_{7/2}) - \epsilon_\nu(0g_{7/2}) \\
 &= [\text{BE}(^{132}\text{Sn}) - \text{BE}(^{133}\text{Sn})] + [\text{BE}(^{132}\text{Sn}) - \text{BE}(^{131}\text{Sn})] \\
 &= 4.84 \text{ MeV}.
 \end{aligned} \tag{5}$$

The first excited negative parity state in ^{132}Sn has an excitation energy of 4.351 MeV and total angular momentum $J^\pi = 3^-$. This indicates that the contribution from the two-body interaction is of the order of ~ 0.5 MeV if we assume a particle-hole model space consisting of the above mentioned single particle states. Compared with closed-shell cores like ^{16}O , ^{40}Ca and ^{56}Ni , the cross-shell interaction for the present study is much weaker, almost an order of magnitude compared with a similar analysis for ^{16}O (in ^{16}O the shell gap for neutrons is 11.521 MeV and the first excited negative parity state $J^\pi = 3_1^-$ is located at 6.129 MeV). This fact applies to the matrix elements of the derived two-body interaction as well, an expected feature since the valence neutrons are on average further apart from each other. An investigation of such trends of the effective interactions for nuclei from ^4He to ^{208}Pb was done in Ref. [29]. There the authors demonstrated that the interaction matrix elements for valence particle systems become on average smaller with increasing mass number A .

In the analysis of the pairing content of our results, we will see that the interplay between the single-particle spacing in ^{133}Sn and the strength of the two-body interaction allows a qualitative understanding of the reported results.

III. RESULTS AND DISCUSSION

In this section we present results of the shell-model calculations for Sn isotopes with up to 10 valence neutrons beyond the ^{132}Sn closed-shell core. As the experimental data available at the present time for the $A \geq 134$ tin isotopes are quite sparse, we have a limited possibility to check the validity of our effective interaction in this region of the nuclear chart. Hence, most of the present results are of a predictive character, providing shell-model guidelines for future experiments of the $A \geq 134$ tin isotopes.

A. Even tin isotopes

The results of the shell-model calculation for the even tin isotopes are displayed in Table II for selected states. We obtain a rather good agreement with the measured energies of the positive parity yrast states in ^{134}Sn [8, 30]. For ^{142}Sn we have calculated only the positive parity states. For this nucleus the number of basis states becomes very large (see Table I), mainly due to the contributions from the $0i_{13/2}$ negative parity orbital. As a consequence, the shell model calculations for ^{142}Sn are rather time consuming. However, if one puts limits on the number of particles which can occupy a particular orbital, the number of basis states can be substantially reduced. Thus, for this nucleus we choose the upper limit of particle number for the $0h_{9/2}$, $1f_{5/2}$ and $0i_{13/2}$ orbitals to be respectively ly 4, 2 and 2. This reduction allows the calculations to be performed in a reasonable time, while introducing only a negligible error to the energies of the nuclear states of ^{142}Sn .

TABLE II: Selected low-lying states for ^{134}Sn , ^{136}Sn , ^{138}Sn , ^{140}Sn and ^{142}Sn . Experimental data for ^{134}Sn are taken from [8, 30]. All entries in MeV.

^{134}Sn			^{136}Sn		^{138}Sn		^{140}Sn		^{142}Sn	
J_i^π	Theory	Experiment	J_i^π	Theory	J_i^π	Theory	J_i^π	Theory	J_i^π	Theory
0_1^+	0.0000	0.0000	0_1^+	0.0000	0_1^+	0.0000	0_1^+	0.0000	0_1^+	0.0000
0_2^+	2.2822		0_2^+	1.8779	0_2^+	1.5280	0_2^+	1.2256	0_2^+	1.0568
1_1^+	2.4141		1_1^+	1.9571	1_1^+	2.0327	1_1^+	2.0539	1_1^+	1.6878
2_1^+	0.7748	0.7256	2_1^+	0.7339	2_1^+	0.7615	2_1^+	0.8028	2_1^+	0.7457
2_2^+	1.6601		2_2^+	1.4642	2_2^+	1.2840	2_2^+	1.3293	2_2^+	1.2221
3_1^+	2.0978		3_1^+	1.8183	3_1^+	1.6072	3_1^+	1.5394	3_1^+	1.5687
4_1^+	1.1161	1.0734	4_1^+	1.1614	4_1^+	1.3550	4_1^+	1.4381	4_1^+	1.5057
4_2^+	1.9486		4_2^+	1.3332	4_2^+	1.5171	4_2^+	1.6308	4_2^+	1.5620
5_1^+	2.1708		5_1^+	1.6877	5_1^+	1.8158	5_1^+	1.7667	5_1^+	1.7471
6_1^+	1.2582	1.2474	6_1^+	1.3770	6_1^+	1.5288	6_1^+	1.8969	6_1^+	1.8155
6_2^+	2.6215		6_2^+	2.1620	6_2^+	2.1359	6_2^+	2.0722	6_2^+	2.0019
7_1^+	2.9538		7_1^+	2.2915	7_1^+	2.1221	7_1^+	2.1126	7_1^+	2.0791
8_1^+	2.4634	2.5089	8_1^+	2.1189	8_1^+	2.3606	8_1^+	2.4042	8_1^+	2.3835
8_2^+	4.6571		8_2^+	2.4125	8_2^+	2.7503	8_2^+	2.6369	8_2^+	2.7592

We have omitted negative parity states. In our case they are constructed from particle excitations and the first states appear at $\sim 4 - 5$ MeV. This is also a region where we expect, from the shell gap of Eq. (5), negative parity particle-hole excitations to appear.

Of interest here is the fact the $0_1^+ - 2_1^+$ spacing remains nearly constant, except for a small increase at ^{140}Sn due to the filling of the $1f_{7/2}$ single-particle orbit. This constancy is similar to what we have for the lighter even tin isotopes, from ^{102}Sn to ^{130}Sn . The difference is that the gap is smaller by approximately 0.5 MeV. Compared with the matrix elements for the model space of the lighter tin isotopes (the model space for ^{102}Sn to ^{130}Sn consists of the single-particle orbits $0g_{7/2}$, $2s_{1/2}$, $0h_{11/2}$, $1d_{3/2}$, $1d_{5/2}$), the interaction matrix elements are on average smaller for the present model space. This is simply due to the fact that the nucleons are farther apart on average from each other than they are for the lighter tin isotopes. For the $0_1^+ - 2_1^+$ spacing, the most important configurations are those where the single-particle orbits $2p_{3/2}$ and $1f_{7/2}$ are involved while the $0h_{9/2}$ plays a role in excited states with higher spin values, as expected from the single-particle spectrum of Fig. 1. Actually, the gross features of both the even and odd low-energy spectrum can be captured by an interaction defined for a model space including only the $2p_{3/2}$ and $1f_{7/2}$ orbits. This feature is similar to what we have in the calcium isotopes, except that the $1p_{3/2}$ and $0f_{7/2}$ orbits have one node less. These properties of the calcium isotopes were discussed more than forty years ago, see for example Refs. [31, 32].

The matrix elements which involve the $2p_{3/2}$ and $1f_{7/2}$ orbits oscillate in absolute value between $\sim 0.5 - 1.0$ MeV. This means, if we adopt a simple pairing model analysis, as done in Ref. [33], that the relation between the single-particle spacing and the interaction matrix elements is of the order of $\sim 0.5 - 1.0$. Within the framework of a simple pairing model, this is a region where we expect pairing correlations to play a dominant role. In this case, the interaction determines much of the pairing gap, viz the difference between the seniority zero ground state and the first excited seniority two state, where we have one broken pair. As we will show below, a generalized seniority analysis indicates indeed that pairing correlations are still strong resulting in a nearby constant $0_1^+ - 2_1^+$ spacing, as is the case for the lighter Sn isotopes. See also the discussion in Ref. [33].

The results for the $B(E2)$ calculations are shown in table III. The values of calculated $B(E2)$ transition probabilities are presented in Weisskopf units (W.u.), defined as [34]

$$B_w(E\lambda) = \frac{(1.2)^4}{4\pi} \left(\frac{3}{\lambda + 3} \right)^2 A^{\frac{2\lambda}{3}} e^2 fm^4, \quad (6)$$

with $\lambda = 2$ in our case. Experimental values of $B(E2)$ for the nuclei of interest are currently available only for two transitions in ^{134}Sn : $B(E2 : 6_1^+ \rightarrow 4_1^+) = 36 \pm 7 \text{ e}^2 \text{fm}^4$ or $0.88 \pm 0.17 \text{ W.u.}$ [35] and $B(E2 : 0_1^+ \rightarrow 2_1^+) \approx 295.0 \text{ e}^2 \text{fm}^4$ or 7.24 W.u. [2]. These transitions probabilities can be reproduced in our calculations by adjusting the effective neutron charge value to $e_n^{eff} = 0.66 e$ for the $6_1^+ \rightarrow 4_1^+$ transition and $e_n^{eff} = 0.62 e$ for the $0_1^+ \rightarrow 2_1^+$ transition, respectively.

One could derive state dependent effective charges along the same lines as for the effective interaction. In this work we limit however ourselves to a constant effective charge. An average of the neutron effective charge obtained above, $e_n^{eff} = 0.64 e$, has been used for every calculated transition. It should be noted that the authors of Ref. [8] use a similar value, $e_n^{eff} = 0.70 e$.

The $B(E2 : 2_1^+ \rightarrow 0_1^+)$ value scale nicely with the number of valence neutrons, except for ^{142}Sn , where we see a reduction due to the weak shell closure at ^{140}Sn , see the discussion below on odd isotopes as well. We notice also

TABLE III: Selected $B(E2)$ values for ^{134}Sn , ^{136}Sn , ^{138}Sn , ^{140}Sn and ^{142}Sn . $e_n^{eff} = 0.64e$. All entries are in W.u.

Transition	^{134}Sn	^{136}Sn	^{138}Sn	^{140}Sn	^{142}Sn
$2_1^+ \rightarrow 0_1^+$	1.50	2.94	4.19	5.72	5.55
$4_1^+ \rightarrow 2_1^+$	1.53	2.04	0.006	1.71	0.78
$6_1^+ \rightarrow 4_1^+$	0.83	0.33	0.27	0.54	0.06
$8_1^+ \rightarrow 6_1^+$	0.12	1.25	0.03	0.08	0.04

that the $B(E2 : 4_1^+ \rightarrow 2_1^+)$ value almost vanishes at ^{138}Sn . From the seniority discussion of subsection III C, we would expect this value to be large since the seniority $\nu = 2$ overlaps with the shell-model wave functions are large for the 2_1^+ and 4_1^+ . Within the framework of a naive seniority picture the corresponding transition between two seniority $\nu = 2$ states is expected to be larger than the value listed in Table III. There are however other contributions as well which cancel out the naive seniority picture. A further analysis and interpretation of the structure of the wave functions is made in connection with our generalized seniority analysis in subsection III C.

B. Odd isotopes

The results of the shell-model calculation for the odd isotopes are displayed in Fig. 2. In our pairing analysis below we show that, except for the ground states and the lowest-lying $3/2^-$ states, most of the other states displayed in Fig. 2 deviate from a one-quasiparticle picture. We postpone therefore an analysis of the odd isotopes to the seniority analysis below, see subsection III C. In connection with the seniority analysis, we would however like to guide the eye to the evolution of the $(1/2)^-$, $(3/2)^-$, $(5/2)^-$, $(7/2)^-$ and $(9/2)^-$ states as functions of A . These states can be well represented by a one-quasiparticle picture, and the energies of these states plunge down in the spectrum with increasing A , a feature shown in Fig. 2. States with a one-quasiparticle character are shown with bold values for the energies. Other states, such as the first $(1/2)^-$ and $(5/2)^-$, which also plunge down in the spectrum with increasing A , are more fragmented in ^{135}Sn and ^{137}Sn , as discussed in subsection III E. The $(11/2)^-$ and $(15/2)^-$ states are more complex many-body states.

For the sake of completeness, we display also selected calculating the $B(E2)$ transition probabilities from the first excited state to the ground state in Table IV. We have used the effective neutron charge value of $e_n^{eff} = 0.64 e$. In the calculations of ^{141}Sn we limited the number of particles which can occupy the $0i_{13/2}$ orbital to 9. The shell closure in ^{140}Sn is reflected in these transitions as well.

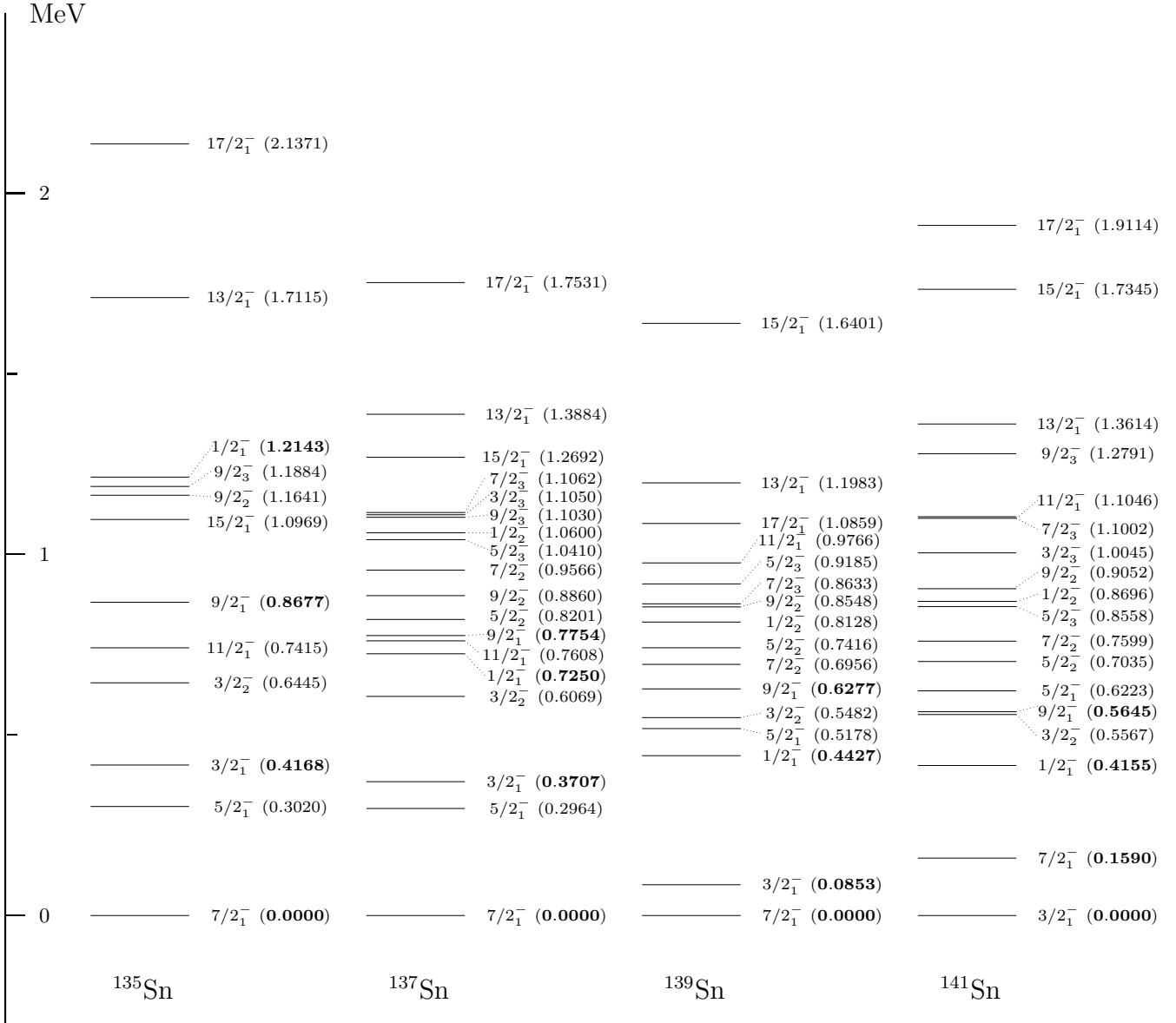


FIG. 2: Selected low-lying states of ^{135}Sn , ^{137}Sn , ^{139}Sn and ^{141}Sn . All entries are in MeV. Boldfaced numbers represent states that develop or have a large one-quasiparticle content. For energies below 1.5 MeV we list up to three states with the same spin for $J^\pi = 1/2^-, 3/2^-, 5/2^-, 7/2^-, 9/2^-$, since these are also discussed in connection with our seniority analysis. For higher spins we list only the Yrast states.

TABLE IV: $B(E2)$ for ^{135}Sn , ^{137}Sn , ^{139}Sn and ^{141}Sn for the lowest states. $e_n^{eff} = 0.64e$. All entries are in W.u.

Transition	^{135}Sn	^{137}Sn	^{139}Sn	^{141}Sn
$(3/2)_1^- \rightarrow (7/2)_1^-$	1.66	0.27	0.005	
$(7/2)_1^- \rightarrow (3/2)_1^-$				0.16

C. Generalized seniority

We will here interpret the shell model results in terms of the generalized seniority model [16]. The generalized seniority scheme is an extension of the seniority scheme, that is from involving only one single orbital in j , the model is generalized to involve a group of j -orbitals within a major shell. The generalized seniority scheme is a simpler model than the shell model since a rather limited number of configurations with a strictly defined structure are included. These states can also be given a straightforward physical interpretation since they represent only selected correlations. If we by closer investigation and comparison of the shell-model wave function and the seniority states find that the most important components are accounted for by the seniority scheme, we can benefit from this and reduce the shell-model basis. This would be particularly useful when we want to do calculations of systems with a large number of valence particles. For an even system, states with seniority $\nu = 0$ are by definition states where all particles are paired. Seniority $\nu = 2$ states have one pair broken, seniority $\nu = 4$ states have two pairs broken, etc. The admixture of seniority $\nu = 4$ states into seniority $\nu = 0$ and $\nu = 2$ states was analysed by for example [17, 18]. It was found that for some states the admixture from seniority $\nu = 4$ states could be as large as 20% .

Outside the closed shell, in our case ^{132}Sn , we construct a generalized seniority $\nu = 0$ pair

$$S^+ = \sum_j \frac{1}{\sqrt{2j+1}} C_j \sum_{m \geq 0} (-1)^{j-m} a_{jm}^+ a_{j-m}^+ \quad (7)$$

and identify the coefficients C_j in the two-particle wave function $S^+|0\rangle$ with the calculated ground state amplitudes of the complete shell-model state ^{134}Sn . Similarly, the generalized seniority $\nu = 2$ operators take the form

$$D_{\mu J}^+ = \sum_{j \leq j', m \geq 0} \frac{1}{\sqrt{(1 + \delta_{j,j'})}} \beta_{j,j'}^{\mu J} \langle jm j' -m | J0 \rangle a_{jm}^+ a_{j'-m}^+ \quad (8)$$

where the coefficients $\beta_{j,j'}^{\mu J}$ are obtained by identifying the two-particle wave function $D_{\mu J}^+|0\rangle$ with the calculated amplitudes of the excited states of ^{134}Sn . Note that excited $J = 0^+$ states are interpreted as generalized seniority $\nu = 2$ states.

In this framework we calculate the even n -particle model states as seniority $\nu = 0$: $(S^+)^{n/2}|0\rangle$ and as seniority $\nu = 2$: $D_{\mu JM}^+(S^+)^{(n-2)/2}|0\rangle$ states. The odd n -particle model states are calculated as seniority $\nu = 1$ $a_{jm}^+(S^+)^{(n-1)/2}|0\rangle$ states. The squared overlaps between the constructed generalized seniority states and our complete shell model states, labelled SM are given in the equations below.

For the even Sn isotopes we evaluate the seniority $\nu = 0$ and $\nu = 2$ square overlaps

$$\mathcal{O}_{J=0}^{\nu=0} = \left| \langle {}^A\text{Sn}(SM); 0^+ | (S^+)^{\frac{n}{2}} | \tilde{0} \rangle \right|^2 \quad (9)$$

and

$$\mathcal{O}_{J_i}^{\nu=2} = \left| \langle {}^A\text{Sn}(SM); J_i | D_{J_i}^+(S^+)^{\frac{n-2}{2}} | \tilde{0} \rangle \right|^2 \quad (10)$$

where the $D_{\mu J}^+$ operator corresponding to the lowest state of a given J in ^{134}Sn has been used. For the odd Sn isotopes, the seniority square overlaps are evaluated according to

$$\mathcal{O}_{J_i}^{\nu=1} = \left| \langle {}^A\text{Sn}(SM); J_i | (S^+)^{\frac{n-1}{2}} a_{J_i \frac{1}{2}}^+ | \tilde{0} \rangle \right|^2 \quad (11)$$

The vacuum state $|\tilde{0}\rangle$ is the ^{132}Sn core, n is the number of valence neutrons and the index i labels the different shell-model states with the same J . These overlaps are thus a measure of the strength of generalized pairing correlations of either the ground state or various excited states.

D. Even isotopes

A typical feature of the seniority scheme is that the spacing of energy levels are independent of the number of valence particles. Our results for the even-mass tin isotopes, see Table II, show that the spacing between the ground state and the 2_1^+ state is fairly constant throughout the whole sequence of isotopes. This suggests that the 2_1^+ states may be described as one broken pair (seniority $\nu = 2$) upon a ground state condensate of 0^+ pairs.

TABLE V: The seniority $\nu=0$ squared overlap $\mathcal{O}_{J=0}^{\nu=0}$ for the ground states and seniority $\nu=2$ squared overlaps $\mathcal{O}_{J_i}^{\nu=2}$ for the lowest-lying eigenstates of the even Sn-isotopes.

J_i^π	^{136}Sn	^{138}Sn	^{140}Sn	^{142}Sn
0_1^+	0.967	0.929	0.879	0.758
2_1^+	0.889	0.824	0.743	0.549
4_1^+	0.586	0.728	0.251	0.251
4_2^+	0.314	0.000	0.174	0.046
6_1^+	0.896	0.702	0.657	0.003
6_2^+	0.000	0.000	0.014	0.410
8_1^+	0.002	0.678	0.573	0.430
8_2^+	0.843	0.001	0.023	0.016

The results displayed in Table V indicate that pairing effects are likely to be important. The 0^+ ground states and the first excited 2^+ states have indeed large seniority $\nu = 0$ and $\nu = 2$ components, respectively. Not unexpectedly, however, these components are gradually decreasing as the number of valence neutrons is increasing. This suggests that the pairing effects become less dominant as one approaches the neutron drip line. A slight exception is seen for the lowest 4^+ states, for which the pairing strength is strongly fragmented in ^{136}Sn , then concentrated again in ^{138}Sn before being further fragmented in ^{140}Sn and ^{142}Sn .

Our calculations also show that (not listed in the above table), in all even isotopes considered, one of the low-lying 0^+ excited states contains a relatively large seniority $\nu = 2$ component.

Note that in evaluating the overlaps of Eqs. (9) and (10) for Table V we used S^+ and D^+ operators determined from the two-particle wave functions of ^{134}Sn and kept constant throughout the shell. This is consistent with both our many-body shell-model approach and the general seniority approach of Talmi. Thus, we have refrained from adjusting the coefficients of Eqs. (9) and (10) as valence pairs are added, despite the fact that this could have given larger overlaps beyond ^{136}Sn .

It is also interesting to point out the behavior of the two 8^+ states listed in Table II. The first 8^+ state in ^{134}Sn is mainly $0h_{9/2}1f_{7/2}$ while the second 8^+ state in ^{134}Sn is almost purely $(0h_{9/2})^2$ and comes at a much higher energy. The lowest 8^+ changes character in ^{136}Sn to a state dominated by the $(1f_{7/2})^4$ configuration, which in a seniority language corresponds to a seniority $\nu = 4$ state. It is first in ^{138}Sn that seniority $\nu = 2$ has a large overlap with the lowest shell-model state, with the configuration $(1f_{7/2})^30h_{9/2}$ as the dominating one.

E. Odd isotopes

For the odd-mass Sn isotopes the seniority $\nu = 1$ square overlaps of Eq. (11) are displayed in Table VI. These quantities allow us to see how the strengths of the various single-quasiparticle levels of ^{133}Sn evolve as valence neutrons are added. We observe that the single-particle strength is largely preserved for the low-lying $9/2^-$, $7/2^-$ and $3/2^-$ states, although being slowly attenuated with increasing number of valence nucleons. For higher lying states such as the $5/2^-$ states, the strength is more strongly fragmented due to mixing with neighboring states. Comparing Table VI with Fig. 2 we furthermore see that the higher single-quasiparticle levels are shifted downwards as neutron pairs are being added in the $f_{7/2}$ level, signaling again the importance of pair correlations in these nuclei. Thus, the $3/2^-$ quasiparticle state, which is at 0.64 MeV in ^{135}Sn and at 0.37 MeV in ^{137}Sn becomes nearly degenerate with the $7/2^-$ ground state in ^{139}Sn and takes over as ground state in ^{141}Sn . For the $9/2^-$ states, we notice that for ^{135}Sn , it is the second excited state which has the largest seniority $\nu = 1$ overlap. For $1/2^-$ there is a lowest state with a large one-quasiparticle overlap except for ^{137}Sn .

The large one-quasiparticle overlaps are also reflected in the fact that the energies of the corresponding states come down in energy as A increases, a feature displayed in Fig. 2 with boldfaced numbers for the states with a large one-quasiparticle content.

TABLE VI: The seniority $\nu=1$ squared overlaps $\mathcal{O}_{J_i}^{\nu=1}$ for the lowest-lying eigenstates of the odd Sn-isotopes.

J_i^π	^{135}Sn	^{137}Sn	^{139}Sn	^{141}Sn
$7/2_1^-$	0.966	0.896	0.813	0.729
$3/2_1^-$	0.005	0.773	0.709	0.689
$3/2_2^-$	0.936	0.088	0.120	0.066
$9/2_1^-$	0.003	0.019	0.580	0.552
$9/2_2^-$	0.176	0.608	0.007	0.002
$9/2_3^-$	0.560	0.000	0.004	0.040
$1/2_1^-$	0.631	0.277	0.368	0.660
$1/2_2^-$	0.223	0.341	0.318	0.018
$1/2_3^-$	0.133	0.138	0.083	0.030
$5/2_1^-$	0.006	0.002	0.008	0.168
$5/2_2^-$	0.272	0.030	0.369	0.357
$5/2_3^-$	0.041	0.308	0.012	0.021
$5/2_4^-$	0.172	0.004	0.005	0.003

IV. CONCLUSIONS

We have performed extensive shell model studies of the heavy tin isotopes with up to ten valence neutrons outside the closed ^{132}Sn core. A realistic microscopic effective interaction has been derived from a modern meson exchange NN potential using many-body perturbation theory. A generalized seniority analysis was applied in order to get insights on the pairing structure of the shell model states. The results of our calculations are in good agreement with the few available experimental data. The other results can serve as a guideline for future experiments. Furthermore, a seniority analysis shows that pairing correlations are strong even for systems with ten valence particles, although the strength of the pairing correlations gets reduced as we increase the number of valence neutrons. Our results, especially for even nuclei, demonstrate that a seniority approach is a good starting point for shell-model studies of heavy Sn isotopes, as observed for the lighter tin isotopes from ^{102}Sn to ^{130}Sn as well.

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- [1] P. Bhattacharyya *et al*, Phys. Rev. Lett. **87**, 062502 (2001).
 - [2] D. C. Radford *et al*, Phys. Rev. Lett. **88**, 222501 (2002).
 - [3] HRIBF newsletter, July 2003, <http://www.phy.ornl.gov/hribf/usersgroup/news/jul-03/jul-03.html#B>
 - [4] I. Dillman *et al*, Phys. Rev. Lett. **91**, 162503 (2003).
 - [5] G. Jacob *et al*, Phys. Rev. C **65**, 024316 (2002).
 - [6] J. Terasaki, J. Engel, W. Nazarewicz, and M. Stoitsov, Phys. Rev. C **66**, 054313 (2002).
 - [7] N. J. Stone *et al*, Phys. Rev. Lett. **94**, 192501 (2005).
 - [8] J. Shergur *et al*, Phys. Rev. C **65**, 034313 (2002).
 - [9] J. Shergur *et al*, Phys. Rev. C **72**, 024305 (2005).
 - [10] J. Shergur *et al*, Phys. Rev. C **71**, 064321 (2005).
 - [11] Proposal CERN-INTC-2003-04.
 - [12] P. Hoff *et al*, Phys. Rev. Lett. **77**, 1020 (1996).
 - [13] L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C **65**, 051306 (2002).
 - [14] F. Hofmann and H. Lenske, Phys. Rev. C **57**, 2281 (1998).
 - [15] J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn, and J. Decharge, Phys. Rev. C **53**, 2809 (1996).
 - [16] I. Talmi, *Simple Models of Complex Nuclei*, Contemporary Concepts in Physics (Harwood Academic Publishers, Chur, Switzerland, 1993) Vol. 7.
 - [17] G. Bonsignori, M. Savoia, K. Allaart, A. van Egmond and G. Te Velde, Nucl. Phys. **A432**, 389 (1985).
 - [18] K. Allaart, E. Boeker, G. Bonsignori, M. Savoia and Y. K. Gambhir, Phys. Rep. **169**, 211 (1988).
 - [19] M. Hjorth-Jensen, T. T. S. Kuo and E. Osnes, Phys. Rep. **261**, 125 (1995).
 - [20] R. Machleidt, F. Sammarruca and Y. Song, Phys. Rev. C **53**, R1483 (1996).
 - [21] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
 - [22] B. A. Brown, N. J. Stone, J. R. Stone, I. S. Towner, and M. Hjorth-Jensen, Phys. Rev. C **71**, 044317 (2005).
 - [23] G. N. White *et al*, Nucl. Phys. **A644**, 277 (1998).
 - [24] A. Holt, T. Engeland, E. Osnes, M. Hjorth-Jensen, and J. Suhonen, Nucl. Phys. **A618**, 107 (1997).
 - [25] R. R. Whitehead, A. Watt, B. J. Cole and I. Morrison, Adv. Nucl. Phys. **9**, 123 (1977).
 - [26] T. Engeland, the Oslo shell model code, 1991-2006, unpublished.

- [27] W. Urban *et al*, Eur. Phys. J. A **5**, 239 (1999).
- [28] K. A. Mezilev, Yu. N. Novikov, A. V. Popov, B. Fogelberg, and L. Spanier, Phys. Scripta T **56**, 272 (1995).
- [29] M. Hjorth-Jensen, H. Mütter, E. Osnes, and A. Polls, J. Phys. **G22**, 321 (1996).
- [30] A. Korgul *et al*, Eur. Phys. J. **A7**, 167 (2000).
- [31] E. Osnes, PhD thesis, University of Oslo, (1966), unpublished.
- [32] T. Engeland and E. Osnes, Phys. Lett. **20**, 424 (1966).
- [33] D. J. Dean and M. Hjorth-Jensen, Rev. Mod. Phys. **75**, 607 (2003).
- [34] A. Bohr and B. R. Mottelson, *Nuclear Structure*, (World Scientific, Singapore, 1998), Vol. **1**.
- [35] C.T. Zhang *et al*, Z. Phys. **A358**, 7 (1997).